WIP: Examining Disparities in Mathematical Literacy within an Asynchronous Online Discussion Community through Core & Periphery & Extra-Periphery Structure

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***Abstract*—The importance of discussing support for learning within online learning communities is widely recognized, yet the diverse user behaviors, especially in the realm of online math learning, are not thoroughly investigated. This work in progress research paper explores the mathematical literacies, and success rates of discussion learning among students taking on different participation roles in an asynchronous online math learning setting. This inquiry is pivotal for comprehending the mechanisms that uphold online learning communities and can provide insights for crafting online discussion activities. This paper employs a mixed-methods approach to analyze big educational data and core-periphery structures within the community. Initially, users are classified into core, periphery, and X-periphery (extra) groups using an extended Surprise detection algorithm, which evaluates interaction quality. The Mann-Whitney U test is applied to assess differences in math literacy and discussion success rates among the groups. The findings reveal that each group is more responsive to its own members, with the core group exhibiting a more balanced response pattern. Notably, the X-periphery group shows the highest success rate in discussions, suggesting that lower activity levels do not compromise communication efficiency. Statistical analysis indicates that while the core and periphery groups have similar levels of math literacy, the X-periphery group excels in all three types of mathematical literacy assessed. These results highlight the importance of considering group dynamics and participation roles when designing online math learning activities to foster effective communication and support within the community. The study's insights into social support traffic offer practical implications for practitioners aiming to enhance the sustainability of online learning communities through tailored discussion activities.**

***Index Terms*—online math learning, community roles, mathematical literacy, educational data analysis**

I. Introduction

Mathematical literacy is the fundamental capacity to apply mathematical concepts and knowledge to address real-world issues and engage in collaborative problem-solving with others [1]. One effective strategy for enhancing mathematical literacy is through mathematical discussion [2]. Learners, through mathematical discourse, communicate potential solutions, substantiate their reasoning, and develop crucial mathematical ideas [3], [4].

The core-periphery model offers insight into online community dynamics, particularly in asynchronous online discussions. It distinguishes between core participants, who drive discussions, and peripheral participants, who engage less but still add diverse perspectives [5]. In educational settings, core participants are vital for stimulating active dialogue and deeper discussions, while peripheral participants contribute occasional but valuable insights. The main challenge is motivating peripheral members to engage more actively to enrich the overall learning experience [6].

Although the significance of online communities in education has been extensively documented in previous studies, scant research has been conducted to comprehend how students with varying levels of math literacies engage in discussion, especially from the perspective of core-periphery structure [7], [8]. Distinguishing user behavior patterns based on social network and activities holds significant importance for the analysis and design of online communities [9], [10]. Conceptually, a core-periphery structure describes students' roles in a social network, where core members tend to have frequent connections with themselves and others in a network. In contrast, periphery members tend to interact with core members but are more loosely connected with other periphery members. More details of the structure are discussed in the Methods section.

To shed light on possible mechanisms to sustain an online learning community, this study aimed to understand the social support dynamics among students in an online math learning environment using educational big data and core-periphery structures. Specifically, we extended a core-periphery detection algorithm [11], [12] by considering interaction quality to classify students with three classes: core, periphery, and X-periphery. Subsequently, statistical analysis was employed to explore the math literacies characteristics across three groups, accompanied by a discussion on their implications for online communities. Specifically, we addressed the following research questions:

RQ1: How can we identify core, periphery, and X-periphery students in the asynchronous online discussion (AOD) community?

RQ2: How do the discussions of students from core, periphery, and X-periphery groups compare in quality regarding math literacy and discussion success rates?

II. Data Context and Collection

We utilized a learning platform that supports asynchronous discussions from Math Nation [13], an online mathematics resource for middle and high school students. In each discussion thread, a student poses a question, and other students or teachers collaborate to discuss and answer the question. To extract high-quality threads, we selected threads from a pool of 408,351 threads, following the standard of at least one student making a post, replying at least once, and having at least one teacher serving as an expert tutor and one student as a peer tutor participating in the discussion. We extracted 2,318 high-quality threads, comprising a total of 24,116 clauses.

To analyze students' discussion qualities, we evaluated and labeled how each clause reflected their math literacy. We employed manual coding to assign binary labels (0 or 1) to each student's post in the thread, indicating their proficiency in the three types of mathematical literacy and discussion success. To explore the efficiency of learning across different groups, it is necessary to annotate each thread with a success label, indicating whether the question has been successfully answered. This annotation is carried out manually by four doctoral students on 257 threads simultaneously. The Inter-rater reliability (IRR) scores among the four annotators were 0.8758 for students' knowledge representation and 0.9202 for discussion success. Subsequently, each annotator individually labels the remaining threads. The mathematical literacy in RQ1 is based on a complete dataset of 52,150 student users to facilitate a better comparison of differences between various groups. The RQ2 focus on a curated dataset of high-quality discussion data, which ensure that students posting have at least one self-reply, one reply from another student, and one reply from the teacher.

In each post, students exhibit three types of mathematical literacy. Firstly, in terms of knowledge and content [14], students demonstrate an understanding of mathematical concepts and provide correct answers. Secondly, regarding competency and skills [15], students display the ability to communicate using mathematical terms and formulas. Lastly, in terms of application in the real world [16], students connect mathematical concepts to explain phenomena in the real world. The success rate of discussions pertains to whether students have successfully learned from the current discussion. When it comes to student-based analysis, mean statistics are utilized to gauge mathematical literacy in each individual post. Furthermore, both original posts and their accompanying replies are counted to assess the success rate of discussions.

III. Methods

To address RQ1, we adapt the Surprise detection algorithm to assess interaction quality and categorize discussion participants into core, periphery, and a new group called X-periphery, refining the traditional binary core-periphery model [7]. This ternary classification recognizes the importance of binary relationships as indicators of interaction quality within online communities [9], [10]. Using the Surprise algorithm, known for its effectiveness in social network analysis [11], [12], [17], we first identify significant core-periphery structures and then differentiate students lacking bidirectional interactions into periphery or X-periphery, thus enhancing our understanding of engagement in student forums.

Specifically, this algorithm estimates the probability distributions of edges and nodes to identify the most probable participant clusters. Following these procedures, students were grouped into three categories: core, periphery, and X-periphery. This classification accounts not only for the frequency and extent of student interactions but also considers the quality and persistence of these interactions by acknowledging binary relationships. The core group includes students with frequent, diverse, high-quality engagements that drive further discussions. The periphery group consists of students with less frequent or more isolated interactions, lower engagement quality, and limited capacity to initiate additional discussions. Lastly, the X-periphery group represents periphery students without binary relationships, signifying minimal and short-lived interactions.

For RQ 2, as described above, groups are derived from the method described in RQ1, and the success rates of three mathematical literacies and successful mathematical discussions are annotated manually by four annotators, while aggregated by user as mean values. Statistical testing is conducted to analyze whether the three math literacies corresponding to the three groups in the dataset exhibit significant differences. Significance testing is performed using the Mann-Whitney U test, a method that does not necessitate adherence to a normal distribution or homogeneity of variance. All methods are described in Figure 1.

IV. Results and Discussions

A. RQ1: Core, Periphery, and X-Periphery Classification

The dataset comprises a total of 2,318 discussion threads with 3,174 participating users, and it includes 27,514 directed edges. Each post or reply is issued by either a student or a teacher. Each discussion thread has one post, and there may be multiple replies. After applying the algorithm described in this article for grouping, relevant descriptive statistics are obtained. These statistics calculate the differing numbers of users, total discussions, and post counts across the three groups in the discussion in Table I.

From table I, it can be observed that the number of core users is relatively small, while the periphery and X-periphery user counts are comparable. However, a notable difference is evident in the average discussion activity. In terms of activity of participation in discussions, the order is Core >Periphery >X-periphery. The disparity in average activity among these three groups indicates that the core actively engages in discussions, whereas the X-periphery demonstrates the least

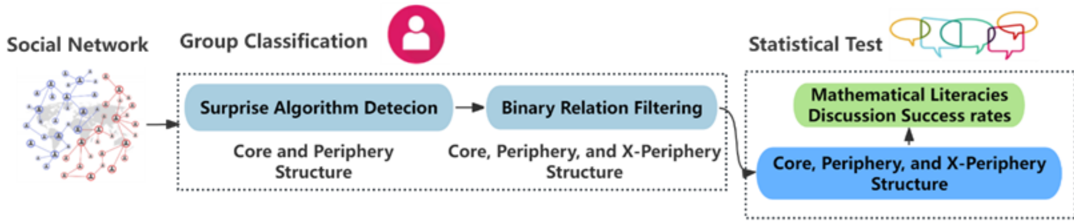

Fig. 1. The method for this work revolves around structural extraction as its core, coupled with relevant attribute analysis.

TABLE I
COUNT THE NUMBER OF USERS FOR DIFFERENT TYPES, AS WELL AS THE QUANTITY OF THEIR PARTICIPATION IN DISCUSSIONS AND THE NUMBER OF POSTS THEY HAVE MADE.

Type/Topic	Core	Periphery	X-periphery
Sum(user)	435	1383	1356
Sum(discussion)	8488	11351	7675
Sum(post)	1061	616	641
Mean(discussion)	19.51	8.20	5.66
Mean(post)	2.43	0.44	0.47

engagement. Regarding posting behavior, the core exhibits the highest frequency, while the periphery and X-periphery are similar. This suggests that the core initiates topics actively, serving as guides for the forum, while both the periphery and X-periphery exhibit significantly fewer proactive posting behaviors compared to the core. The core group students participate extensively in online discussions, actively engaging with peripheral students. core group students tend to help sustain the online community by providing more supportive information than other students, despite their smaller numbers.

Continuing with the statistics of different reply roles for each poster as shown in Table II, when the poster belongs to one of the three groups, it is primarily determined by whom the replies are made, taking into account reply intensity, where the linear weight percentage of multiple replies will be greater.

TABLE II
THE PROPORTION OF REPLIES FROM DIFFERENT GROUPS CORRESPONDING TO POSTS FROM DIFFERENT GROUPS. (THE VERTICAL SUM IN THE TABLE IS 100%)

Role/Type	Core	Periphery	X-periphery
Core	66.03%	18.18%	0.13%
Periphery	31.97%	75.84%	21.74%
X-periphery	1.98%	5.97%	76.89%

From table II, it's evident that each group tends to respond most to posts made by members of their own group. However, the core exhibits a more balanced pattern compared to the periphery, with core's responses containing a higher proportion of contributions from the periphery. Interestingly, the X-periphery demonstrates the highest rate of self-group responses within the group, and it engages more with the periphery rather than the core. This might imply that groups on the periphery and those on the X-periphery would grow closer, fostering more communication between them, while the core would engage in more communication with the periphery.

B. RQ2: Math Literacy and Discussion Success Rates

Performing statistical analysis to determine whether there is a significant statistical difference among three separate groups in terms of three levels of mathematical literacy and their success rate in discussions. A unified Mann-Whitney U non-parametric test is conducted, as shown in Table 3 (Appendix). After conducting the Mann-Whitney U test on multiple groups, multigroup comparison allows for the utilization of multiple comparison correction methods to obtain more stringent p-values in Table III. A commonly used approach is the Bonferroni correction, which divides the original significance level (0.05) by the number of comparisons made. The significance level used for each test is adjusted to ensure that the overall significance level remains within a reasonable range, thereby enhancing the credibility of the results.

Based on the three categories of mathematical literacies, it's evident that in the majority of cases, both the core and the periphery are generally similar, yet the core surpasses the periphery in terms of Competency and skills as well as Application in real-world scenarios. Additionally, it's observed that the X-periphery demonstrates the highest level of performance. In terms of success rate and Knowledge and content, the levels appear to be X-periphery >Periphery >Core. Considering the aforementioned findings, it implies that although the activity level of the Peripheral groups is lower, their success rate in discussions is higher. Moreover, the success rate is highest among the extra-peripheral group, indicating that despite their fewer engagement activities, they pose accurate questions and exhibit high communication efficiency. On the other hand, there is a similar pattern observed across the three types of mathematical literacy, where the most (extra) peripheral group excels in all three types. This reflects why they can sustain effective communication in mathematical discussions. However, the gap in mathematical literacy between the Peripheral groups and the core is not substantial. Despite being slightly lower, their success rate in discussions remains higher than that of core. This suggests that fewer discussion activities among groups may result in maintaining higher communication effectiveness, which isn't solely dependent on their mathematical literacy. Nonetheless, having the highest level of mathematical literacy does aid in communication.

TABLE III
MANN-WHITNEY U STATISTICAL TESTING FOR THE KNOWLEDGE AND CONTENT CORRESPONDING TO DIFFERENT TYPES OF USERS.

Type	Group1	Group2	Mean1 (%)	Mean2 (%)	P value	Adjusted P value
Knowledge and content	Core	Periphery	25.90	24.56	0.001**	0.012*
	Core	X-periphery	25.90	36.91	2.93e-07***	3.51e-06***
	Periphery	X-periphery	24.56	36.91	5.10e-41***	6.12e-40***
Competency and skills	Core	Periphery	10.33	10.03	0.0001***	0.0012**
	Core	X-periphery	10.33	10.89	0.0009***	0.0108*
	Periphery	X-periphery	10.03	10.89	0.782	1.0
Application in the real world	Core	Periphery	8.52	7.77	1.38e-05 ***	1.65e-4***
	Core	X-periphery	8.52	9.03	0.002 **	0.024*
	Periphery	X-periphery	7.77	9.03	0.119	1.0
Success rate	Core	Periphery	36.48	42.65	0.65	1.0
	Core	X-periphery	36.48	52.43	4.41e-07 ***	5.29e-06***
	Periphery	X-periphery	42.65	52.43	6.85e-10 ***	8.22e-09***

*Note. *p<0.05, **p<0.01, ***p<0.001*

These findings suggest that fostering interactions between core and X-periphery students in online communities can bridge gaps and enhance learning. Supporting the selective yet efficient engagement of X-periphery students can benefit the entire community by integrating their insights into broader discussions.

V. Conclusion

This research endeavors to uncover the behavioral disparities among distinct groups, followed by an exploration of the variations in their mathematical literacies based on these different groups. By doing so, it seeks to furnish researchers and practitioners with empirical insights into how an online learning community is sustained with social support.

Several promising directions for future research are worth exploring. First, the detection of the core-periphery structure is currently performed in a post-hoc manner. It would be valuable to examine the longitudinal evolution of this structure and investigate whether social support dynamics shift as cores are formed within the community. Analyzing thematic differences in discussions across different groups could also shed light on the focal points of groups with varying levels of mathematical literacy [18]. Additionally, incorporating automated feedback may help support the development of more effective asynchronous discussion communities [19]. Finally, we aim to extend this study by including students' in-platform assessment results to gain deeper insights into the relationship between the behaviors and mathematical literacies of core, periphery, and X-periphery students.

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